Multifractal Universe)

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In the frame of the multifractal theory of time and space (in this model our universe is consisting of real time and space fields and is the multifractal universe) in the works [1]- [?] some of problems were analyzed: how the fractional dimensions of real fields of time and space influence on behavior of different physical phenomena. In this paper it is shown that in the multifractal theory of time and space the corrections of general relativity to Newton gravitational forces are explaining very simple in Newton approach. It is shown also that there is dependence (though very small) of bodies gravitational acceleration from their masses, so the principle of equivalence in multifractal universe is not absolute and it is only a very good approach. Thus taking into account the multifractal dimensions of time gives small corrections to gravitation forces of Einstein general relativity and explain it in Newton approach. These corrections are equal to $F_g(r_0ma)(Mr_m)^{-1}$ where F_g is the gravitational force originated by mass M at the distance r where the mass m is located, r_0 is the gravitational radius of the mass M and the r_m is the mean radius of the mass m, a is a numerical factor depending of distribution of masses of m . The possibility of experimental checking this effect in weak gravitational fields is analyzed . It makes for perihelion of Mercury rotation the correction to known results of Einstein relativity. This correction is equal $\sim 0,5$ of percent of Einstein corrections.

CONTENTS:

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I. INTRODUCTION

The fractal model of space and time (Kobelev [1]- [16]) treats the time and the space with fractional dimensions as real fields. Some information about the physical model of time and space based on these works we use in this paper. Our universe is formed only by these fields , i.e. universe is fractional real material time and fractional real material space and all other fields are born by the fields of time and space. As the time and the space are material fields with fractional dimensions and multifractal structure (multifractal sets,) they are defined on the sets of their carriers of measure (physical vacuum of our universe). In each the time (or the space) point ("points" are approach for very small "intervals" of time or space and "intervals" are the multifractal sets with global dimensions for their sets, that play role of local dimensions for universe in whole) the fractional dimensions of time (or space) determine the densities of Lagrangians energy for all physical fields (or the new physical fields for space) in these points. Time and space are binding by relation $d\mathbf{r}^2 - c^2dt^2 = 0$ (this relation is only a good approach, more precise relations see at [1]- [2]). As the real fields time and space own by huge supply of energy (the question about their energy was considered partly at [16]) and these energies may be evaluated. The purpose of this paper is more detailed consideration of very important problem (in the frame of mathematical formalism of the multifractal model of time and space introduced in the papers of Kobelev and presented in [1]- [16]): are the dependence of acceleration of any body with mass m at its mass exists or not exists (if acceleration is born by gravitational force)? The general relativity is based on the principle of equivalence and deny the positive answer on this question a priori. A posteriori this problem had not decided till now, because, as it seems, the all experiments had fulfilled till now have small unexplained corrections. For example, the Mercury perihelium rotation has unexplained corrections to general relativity results that consist 0.5. percent of general relativity corrections. The multifractal theory of time and space may explain this correction and gives dependence of acceleration of body at its mass (though very small). Thus the principle of equivalence of general relativity lose its absolute character and become a good approach principle. The reason of it lay in the multifractal nature of time and space, i.e. in the multifractal nature of our universe. Time and space build up the universe and by means of their multifractal dimensions construct the known picture of all physical fields. So the FD of mass of moving body have dependence of gravitational energy (see Kobelev [1]- [3]) and this dependence is not be cancelled even in Newton equation (the latter is the rude approach of scalar gravitation or the registration only the scalar component of tensor gravitational field). The fractional dimensions of time give the corrections to gravitational forces such as corrections of general gravity plus additional correction depending of accelerated bodies masses . We analyze the value of the corrections born by this effect and consider as example the Mercury perihelium rotation. Note that the general relativity postulating of principle of equivalence not be rigorous in the multifractal universe and because of the other simple reason: all systems of reference in real time and space fields of multifractal universe are absolute systems and can be differentiate one of another because of absolute character of time-space fields and connection of system of reference with absolute fields. Only the smallness of fractional corrections to topological time dimension in our domain of universe allows to use the principle of equivalence as a very good approach in our life and science and it works beautiful though it is not rigorous .

II. NEWTON EQUATIONS IN THE MULTIFRACTAL UNIVERSE

In this paragraph we write down the modified Newton equations in the multifractal time universe in the presence of gravitational forces only (see [1]- [3])

$$D_{-,t}^{d_t(r,t)} D_{+,t}^{d_t(r,t)} \mathbf{r}(t) = D_{+,r}^{d_r} \Phi_g(\mathbf{r}(t)) \tag{1}$$

$$D_{-,r}^{d_r} D_{+,r}^{d_r} \Phi_g(\mathbf{r}(t)) + \frac{b_g^2}{2} \Phi_g(\mathbf{r}(t)) = \gamma \tag{2}$$

In the Eq.(2) the constant b_g^{-1} has order of size of universe and is introduced with purpose to extend the class of functions on which the generalized fractional derivatives concept is applicable. These equations are not a closed system because of presence of the fractionality of spatial dimensions. Therefore we approximate the fractional derivatives with respect to space coordinate as $D_{+,\mathbf{r}}^{d_\mathbf{r}} \approx \nabla$, i.e. approximate them by usual space derivatives. In the Eqs. (1)-(2) is used the integral functionals $D_{+,t}^{d_t}$ (both left-sided and right-sided) which are suitable to describe the dynamics of functions defined on multifractal sets (see [1]- [3], [7]). These functionals are simple and natural generalization of the Riemann-Liouville fractional derivatives and integrals and read:

$$D_{+,t}^{d} f(t) = \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}} \tag{3}$$

$$D_{-,t}^{d} f(t) = (-1)^n \left(\frac{d}{dt}\right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n-d(t'))(t'-t)^{d(t')-n+1}} \tag{4}$$

where $\Gamma(x)$ is Euler's gamma function, and a and b are some constants from $[0,\infty)$. In these definitions, as usually for Riemann-Liouville derivatives, $n = \{d\} + 1$, where $\{d\}$ is the integer part of d if $d \geq 0$ (i.e. $n-1 \leq d < n$) and $n = 0$ for $d < 0$. If $d = const$, the generalized fractional derivatives (GFD) (3)-(4) coincide with the Riemann - Liouville fractional derivatives ($d \geq 0$) or fractional integrals ($d < 0$). When $d = n+\varepsilon(t)$, $\varepsilon(t) \to 0$, GFD can be represented by means of integer derivatives and integrals. For $n = 1$, i.e. $d = 1+\varepsilon$, $|\varepsilon| << 1$ it is possible to obtain:

$$D_{+,t}^{1+\varepsilon} f(\mathbf{r}(t),t) \approx \frac{\partial}{\partial t} f(\mathbf{r}(t),t) + \\ +a\frac{\partial}{\partial t}\left[\varepsilon(r(t),t) f(\mathbf{r}(t),t)\right] + \frac{\varepsilon(\mathbf{r}(t),t) f(\mathbf{r}(t),t)}{t} \tag{5}$$

where a is a *constant* and determined by choice of the rules of regularization of integrals ([1]- [2], [7]) (for more detailed see [7]) and the last additional member in the right hand side of (5) is very small (in this member the t in the denominator begins from "big-bang") so when the problem of irreversibility is not researching this member may be omitted . The selection of the rule of regularization that gives a real additives for usual derivative in (3) yield $a = 0.5$ or $a = 1$ for $d < 1$ [1]. The functions under the integral sign in the (3)-(4) we consider as the generalized functions defined on the set of the finite functions or Gelfand functions [21]. The notions of the GFD, similar to (3)-(4), can also be defined and for the space variables $\mathbf{r}$. The definitions of GFD (3)-(4) need in the connections between the fractal dimensions of time $d_t(\mathbf{r}(t),t)$ and the characteristics of physical fields (say, potentials $\Phi_i(\mathbf{r}(t),t)$, $i = 1,2,..$) or densities of Lagrangians L_i) and it was defined in cited works. Following [1]- [15], we define this connection by the relation

$$d_t(\mathbf{r}(t),t) = 1 + \sum_i \beta_i L_i(\Phi_i(\mathbf{r}(t),t)) \tag{6}$$

where L_i are densities of energy (Lagrangian densities) of physical fields, β_i are dimensional constants with physical dimension of $[L_i]^{-1}$ (it is worth to choose β_i' in the form $\beta_i' = a^{-1}\beta_i$ for the sake of independence from the regularization constant and select the $\beta = Mc^2$ where M is the mass of the body that born considered gravitational field). The definition of the time as the system of subsets and definition of the FD for d_t (see (6)) connects the value of fractional (fractal) dimension $d_t(r(t),t)$ with each time instant t. The latter depends both on time

t and coordinates $\mathbf{r}$. If $d_t = 1$ (an absence of physical fields) the set of time has topological dimension equal to unity. We bound consideration only the case when relations $d_t = 1-\varepsilon(\mathbf{r}(t),t))$, $|\varepsilon| \ll 1$ are fulfilled. In that case the GFD (as was shown in [7]) may be represented (as a good approach) by ordinary derivatives and relation (1), (5)) are valid. Now we can determine the d_t for distances much larger than the gravitational radius r_0 (for problem of a body motion in the field of spherical-symmetric gravitating center) as (see [1] for more details)

$$d_t \approx 1 + \beta_g \Phi_g + \beta_g \Phi_m \tag{7}$$

where Phi_g is the gravitational potential born by the mass M in the center of body with mass m, Φ_m is the gravitational potential born by the body with mass m in its center. So the equations (1)-(2) reed (we used for GFD approach of (5)([16]))

$$[1 - 2\varepsilon(\mathbf{r}(\mathbf{t})] \frac{d^2}{dt^2}\mathbf{r} = \mathbf{F}_g \tag{8}$$

where

$$\mathbf{F}_g = -\nabla \frac{\gamma M}{r}, \varepsilon = \beta_g(\Phi_g + \Phi_m) \tag{9}$$

We had neglected by the fractional parts of spatial dimensions and by the contributions from the term with b_g^{-1}. Now we take the β_g as $\beta_g = c^{-2}$ for potentials (or $\beta = (Mc)^{-2}$ for Lagrangian density L). Let the body with mass m moves in the gravitational field of the body with mass M on the distance r ($r >> r_0$) where r_0 is the gravitational radius of the body with mass M) and let $M >> m$. Then it is possible to consider these bodies as points masses. Now we more precisely calculate corrections to the time dimensions in the Eq.(9). At the point r, where is a center of point mass m, there are two contributions in the FD of d_t: one is caused by the field of the body with mass M, another is caused by the mean gravitational field of the body with mass m and with mean radius r_m . The latter is

$$\Phi_m = \frac{am\gamma}{r_m}$$

where a is number factor defining by distribution of mass within body with mass m. This factor is near unity (for uniform distribution it equal 1.5). If we want to describe the Mercury perihelium rotation, than it is necessary to use the law of conservation of energy . From the energy conservation law (now this law is only approximate law (though a very good for $d_t \sim 1$) since our theory and mathematical approaches have used apply only to open systems) obtain

$$\left[1 - \frac{2\gamma M}{c^2 r}(1 + \frac{amr}{Mr_m})\right] \left(\frac{\partial r(t)}{\partial t}\right)^2 - \frac{2mc^2}{r} + \\ + \left[1 - \frac{2\gamma M}{c^2 r}(1 + \frac{amr}{Mr_m})\right] r^2 \left(\frac{\partial \varphi(t)}{\partial t}\right)^2 = 2E \tag{10}$$

Here we used the approximate relation between the generalized fractional derivative and usual integer-order derivative (5),(8) and notations corresponding to conventional description of the motion of mass m near gravitating center M again. The Eq.(10) differs from the corresponding classical limit of the equations of general relativity by the presence of additional terms in the first square bracket of the left-hand part of the equation and the parentheses in the square bracket in the right-hand part of this equation. The first term describes the velocity alteration during gyration and is negligible in the case, when perihelion gyration of Mercury is calculated. If we neglect by it, then the Eq.(10) reduces to the corresponding classical limit of general relativity equation with the correction born by breaking the principle of equivalence. For large energy densities (e.g., gravitational field at $r < r_0$) the Eqs.(1)-(2) contain no divergencies [1] since integral-differential operators of the generalized fractional differentiation reduced to the generalized fractional integrals (see (1)-(4)).

III. IS GRAVITATIONAL ACCELERATION DEPENDS OF THE MASS OF THE ACCELERATED BODY?

In this paragraph we consider the problem of limited validity of the principle of equivalence of the general relativity. It is well known that the principle of equivalence (i.e. the independence of acceleration of body with mass m given by the gravitational field of it mass) is the base of the general theory of relativity. Is this principle rigorous or only the good approach? In the multifractal universe (our universe is non-closed system) all the laws and the principle that rigorous for closed systems are not rigorous and are only the good approaches for domain of universe where physical forces are small and so the fractional corrections to the dimensions of time and space are small too. Now we demonstrate an approximate character of this principle by calculating corrections to gravitational force in the space with fractional dimensions of time for the case of classic limit (Newton equation, see the ([3]). In that case the Eq.(8) reads ($r_0 = \frac{2\gamma M}{c^2}$ and $a = 1,5$)

$$[1 - \frac{r_0}{r}(1 + \frac{amr}{Mr_m})] \frac{d^2}{dt^2}\mathbf{r} = \nabla \frac{\gamma M}{r} = \mathbf{F_g} \tag{11}$$

This equation may be rewritten (if take into account that $r_0 r^{-1} \ll 1$) as

$$\frac{d^2}{dt^2}\mathbf{r} \approx \mathbf{F_g}[1 + \frac{r_0}{r}(1 + \frac{amr}{Mr_m})] \tag{12}$$

The right-hand part of the Eq.(12) describes the gravitation force applied to the body placed in the central gravitational field with the correction of general relativity and the extra correction because of violation of principle of equivalence (the member with dependence of mass m).

IV. EXPERIMENTAL CHECKING

The gravitational force defined by relation (12) may be estimated for different physical phenomena. It gives for effect of Mercury perihelium rotation the additional correction near $0.004a$ of corrections of general relativity and results to full coincidence the experimental data for this effect and theoretical calculations. We stress that in multifractal universe there is dependence of the gravitational acceleration from mass of accelerated body. That dependence is very small in the considered case, but has principal role and shows the limitation of the principle of equivalence. Such dependence is natural for the multifractal universe, as all systems of reference in it are absolute systems. So any equivalence of them is treated as approximation (very good in the domains with almost topological dimensions of time and space). Let us write the relation for relative change of part of acceleration depending of mass the accelerating body

$$\frac{a - F_g}{a} = \frac{r_0}{r}(1 + \frac{3mr}{2Mr_m}) \tag{13}$$

The Eq. (13) allows to estimate the role of breaking of the principle of equivalence for different physical phenomena. It is easy examine that our conclusions do not contradict the experiments in this domain and give small corrections to theoretical calculations of general relativity .

V. CONCLUSIONS

The main results of paper are:
1. It is demonstrated the existence of very small gravitational acceleration in the multifractal universe with dependence of masses of accelerated bodies;
2. Our result testify about of non-absolute character of the equivalence principle of the general relativity, but allows to use it as a very good approximated law for the weak gravitational fields;
3. We pay attention also that for describing the experiments which usually are explained only by the general relativity, in the fractal theory of time and space for for this purpose it is enough the approach of Newton equation in the time with fractional dimensions. Certainly, the consideration of relativistic tensor gravitation fields gives the same results in the classic limit.

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[1] Kobelev L.Ya., Fractal Theory of Time and Space ,Preprint at Dep. in VINITI 19.08.99, No.2677-B99 ,(in Russian)

[2] Kobelev L.Ya., *Fractal Theory of Time and Space*, Konros,1999, p.136(in Russian)

[3] Kobelev L.Ya., What Dimensions Do the Time and Space Have : Integer or Fractional? Preprint at http://arXiv.org/abs/physics/0001035

[4] Kobelev L.Ya., Can a Particle's Velocity Exceeds the Speed of Light in the Empty Space? Preprint at http://arXiv.org/abs/gr-qc /0001042

[5] Kobelev L.Ya., Multifractality of Time and Space, Covariant Derivatives and Gauge Invariance, Preprint at http://arXiv.org/abs/hep-th /0002005

[6] Kobelev L.Ya. Does Special Relativity Have Limits of Applicability in the Domain of Very Large Energies? Preprint at http://arXiv.org/abs/physics/0005069

[7] Kobelev L.Ya.,Generalized Riemann -Liouville Fractional Derivatives for Multifractal Sets,Preprint at http://arXiv.org/abs/math.CA/0002008,;

[8] Kobelev L.Ya.The Theory of Fractal Time: Field Equations (the Theory of Almost Inertial Systems and Modified Lorentz Transformations), Preprint at http://arXiv.org/abs/physics/0005068

[9] Kobelev L.Ya. The Theory of Gravitation in the Space - Time with Fractal Dimensions and Modified Lorentz Transformations, Preprint at http://arXiv.org/abs/physics/0006029

[10] Kobelev L.Ya. Physical Consequences of Moving Faster than Light in Empty space, Preprint at http://arXiv.org/abs/gr-qc /0001043

[11] Kobelev L.Ya. The Multifractal Time and Irriversibility in Dynamic Systems, Preprint at http://arXiv.org/abs/physics/0002002

[12] Kobelev L.Ya. Maxwell Equation, Schrödinger Equation, Dirac Equation, Einstein Equation Defined on the Multifractal Sets of the Time and the Space Preprint at http://arXiv.org/abs/gr-qc/0002003

[13] Kobelev L.Ya. Is it Possible to Transfer an Information with the Velocities Exceeding Speed of Light in Empty Space? Preprint at http://arXiv.org/abs/physics/0002003

[14] Kobelev L.Ya. Are the Laws of Thermodynamics Consequences of a Fractal Properties of Universe? Preprint at http://arXiv.org/abs/physics/0003036

[15] Kobelev L.Ya. Do Electromagnetic and Gravitation Fields Have Rest Masses? Preprint at http://arXiv.org/abs/physics/0006043

[16] Kobelev L.Ya.Why We Can Not Walk To and Fro in Time as Do it in Space? (Why the Arrow of Time is Exists?) Preprint at http://arXiv.org/abs/physics/0011036

[17] Why Wave Functions in Quantum Mechanics must Treat as Probability Amplitudes and What is Nature of Planck Constant? Preprint at http://arXiv.org/abs/.

[18] Klimontovith Yu.L., *Statistical Theory of Open Systems. Vol.1*, Moscow, Yanus, 1995, 686p. (in Russian); Kluwer Academic Publishers, Dordrecht, 1995; Klimontovich Yu.L., *Statistical Physics of Open systems. Vol.2*, Moscow, Yanus, 1999, 450p. (in Rusian).

[19] Mandelbrot B., *Fractal Geometry of Nature*, W.H.Freeman, San Francisco, 1982

[20] Samko S.G, Kilbas A.A., Marithev I.I., *Integrals and Derivatives of the Fractional Order and Their Applications*, (Gordon and Breach, New York, 1993).

[21] I.M.Gelfand, G.E.Shilov, *Generalized functions* (Aca-

demic Press, New York, 1964)